ISRA UNIVERSITY

Islamabad Campus

Semester – Spring 2019

Calculus-II

Assignment – 7 **Due Date: 11/06/2019**
Marks: 30 **Handout Date: 24/05/2019**

Question # 1:

Find the general solution to the following differential equation using separation of variables:

$$y' = xe^{2y}$$

Solution:

$$\frac{dy}{dx} = xe^{2y}$$

Separating the variables:

$$\frac{1}{e^{2y}}dy = xdx$$
$$e^{-2y}dy = xdx$$

Integrating both sides:

$$\int e^{-2y}dy = \int xdx$$
$$-\frac{1}{2}e^{-2y} = \frac{x^2}{2} + c_1$$
$$e^{-2y} = (-2)\frac{x^2}{2} + (-2)c_1, \qquad \therefore -2c_1 = c$$
$$e^{-2y} = -x^2 + c$$

Taking natural log on both sides:

$$\ln(e^{-2y}) = \ln(-x^2 + c)$$
$$-2y = \ln(-x^2 + c)$$
$$y = -\frac{1}{2}\ln(-x^2 + c), \quad General\ solution$$

Question # 2:

Find the general solution of the following equation using method of exact differential equations:

$$2xy\frac{dy}{dx} + y^2 - 2x = 0$$

Solution:

$$(y^2 - 2x)dx + 2xy\,dy = 0$$

Check for exactness:

$$M = y^2 - 2x, \quad N = 2xy$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}[y^2 - 2x] = 2y \ , \quad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}[2xy] = 2y$$

Hence:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

The given differential equation is exact.

The general solution u (x, y) = c is given by:

$$u(x,y) = \int M dx + k(y)$$

$$u(x,y) = \int (y^2 - 2x) dx + k(y)$$

$$u(x,y) = y^2 x - \frac{2x^2}{2} + k(y)$$

$$u(x,y) = y^2 x - x^2 + k(y)$$

Now to find k (y) lets differentiate the above equation:

$$u(x,y) = \frac{\partial}{\partial y}[y^2 x - x^2 + k(y)] = N(x,y)$$

$$u(x,y) = 2yx + k'(y) = 2xy$$

$$k'(y) = 0$$

Now integrate k' (y):

$$\int k'(y)\, dy = \int 0\, dy$$

$$k(y) = c_1$$

Now:

$$u(x,y) = y^2 x - x^2 + c_1$$

The general solution is:

$$y^2 x - x^2 = c$$

Question # 3:

Solve the given Bernoulli's equation:

$$\frac{dy}{dx} + \frac{y}{x} = y^2$$

Solution:

$$\frac{dy}{dx} - P(x)y = Q(x)y^n$$

$$y' - \frac{1}{x} y = y^2 \to (1)$$

Now $a = 2$ then:

$$u = y^{1-a} = y^{1-2} \Rightarrow y^{-1}$$

Now differentiate u:

$$\frac{du}{dy} = -y^{-1-1} = -y^{-2} \; or \; u' = -y^{-2}y'$$

Multiply $-y^{-2}$ with equ. (1):

$$-y^{-2}y' - (-y^{-2})\frac{1}{x}y = (-y^{-2})y^2$$

$$-y^{-2}y' + \frac{1}{x}y^{-2+1} = -y^{-2+2}$$

$$-y^{-2}y' + \frac{1}{x}y^{-1} = -y^0$$

$$-y^{-2}y' + \frac{1}{x}y^{-1} = -1 \rightarrow (2)$$

Now let put $u' = -y^{-2}y', u = y^{-1}$:

$$u' + \frac{1}{x}u = -1 \rightarrow Linearform$$

$$p(x) = \frac{1}{x}, r(x) = -1$$

$$h = \int p(x)dx = \int \frac{1}{x}dx = \ln x$$

Then:

$$u = e^{-h}\left[\int e^h r dx + c\right]$$

$$u = e^{-lnx}\left[\int e^{\ln x}(-1)\, dx + c\right]$$

$$u = e^{-(\ln x)}\left[\int e^{(\ln x)}(-1)\, dx + c\right]$$

Applying exponent rule: $e^{-(\ln x)} = \left(e^{(\ln x)}\right)^{-1}$

$$u = \left(e^{(\ln x)}\right)^{-1}\left[\int e^{(\ln x)}(-x)\, dx + c\right]$$

$$u = x^{-1}\left[\int x.(-1)\, dx + c\right]$$

$$u = \frac{1}{x}\left[\int -x\, dx + c\right]$$

$$u = \frac{1}{x}\left[-\frac{x^2}{2} + c\right]$$

$$u = \frac{1}{x}\left(-\frac{x^2}{2}\right) + \frac{c}{x}$$

$$u = \left(-\frac{x}{3}\right) + \frac{c}{x}$$

Now: $u = y^{-1}, y = u^{-1}$

$$y = \left[\left(-\frac{x}{3}\right) + \frac{c}{x}\right]^{-1}$$

Good Luck