ISRA UNIVERSITY

Islamabad Campus

Semester – Spring 2019

Solution

Calculus-II

Assignment – 6 | **Due Date: 11/06/2019**

Marks: 10 | **Handout Date: 24/05/2019**

Question # 1:

Solve the following Nonhomogeneous Differential equation using the Basic rule:

$$y'' - 2y' - 3y = e^{2t}$$

Solution:

Step#1: General solution of the homogeneous ODE:

$$y'' - 2y' - 3y = 0$$

The characteristic equation will be:

$$\lambda^2 - 2\lambda - 3 = 0$$

Using Quadratic equation:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2}$$

$$\lambda = \frac{2 \pm \sqrt{4 + 12}}{2} = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2}$$

$$\lambda_1 = \frac{2+4}{2} = \frac{6}{2} \Rightarrow 3, \qquad \lambda_2 = \frac{2-4}{2} = \frac{-2}{2} \Rightarrow -1$$

Then the general solution is:

$$y_h = c_1 e^{3t} + c_2 e^{-t}$$

Step#2: Solution y_p of the non-homogeneous ODE:

Let:

$$y = Ae^{2t}, y' = 2Ae^{2t}, y" = 4Ae^{2t}$$

Substitute them back in the given nonhomogeneous ODE:

$$(4Ae^{2t}) - 2(2Ae^{2t}) - 3(Ae^{2t}) = e^{2t}$$

$$4Ae^{2t} - 4Ae^{2t} - 3Ae^{2t} = e^{2t}$$

$$0 - 3Ae^{2t} = e^{2t}, omitting\ e^{2t}\ gives$$

$$-3A = 1 \Longrightarrow A = -\frac{1}{3}$$

$$y_p = -\frac{1}{3}e^{2t}$$

Therefore:

$$y = y_h + y_p = c_1 e^{3t} + c_2 e^{-t} - \frac{1}{3}e^{2t}$$

Question # 2:

Solve the following Nonhomogeneous Differential equation using the Sum rule:

$$3y'' + 27y = 3\cos x + \cos 3x$$

Solution:

Step#1: General solution of the homogeneous ODE:

$$3y'' + 27y = 0$$

The characteristic equation will be:

$$3\lambda^2 + 27 = 0$$

Using Quadratic equation:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(0) \pm \sqrt{(0)^2 - 4(3)(27)}}{2(3)}$$

$$\lambda = \frac{0 \pm \sqrt{-324}}{6}$$

$$\lambda_1 = +\frac{i18}{6} \Rightarrow +i3\,, \qquad \lambda_2 = -\frac{i18}{6} \Rightarrow -i3$$

Then the general solution is:

$$y_h = e^{-0x}[A\cos 3x + B\sin 3x]$$

$$y_h = A\cos 3x + B\sin 3x$$

Step#2: Solution y_p of the non-homogeneous ODE:

Let:

Since $r(x) = 3\cos x + \cos 3x$, use the sum rule:

$$y_p = y_{p1} + y_{p2}$$

Where we will apply the basic rule to find y_{p1} and y_{p2}:

$$y_{p1} \Longrightarrow corresponds\ to\ 3\cos x$$

$$y_{p2} \Longrightarrow corresponds\ to\ \cos 3x$$

$$r_1(x) = 3\cos x \Rightarrow \ y_{p1} = K_1\cos x + M_1\sin x$$

$$r_2(x) = \cos 3x \Rightarrow \ y_{p2} = K_2\cos 3x + M_2\sin 3x$$

y_{p2} happens to be a solution of the homogeneous ODE, so we have to multiply it by x (The Modification rule):

$$y_{p2} = K_2 x\cos 3x + M_2 x\sin 3x$$

$$y_p = y_{p1} + y_{p2} \Rightarrow K_1\cos x + M_1\sin x + K_2 x\cos 3x + M_2 x\sin 3x$$

$$y'_p = -K_1\sin x + M_1\cos x + K_2\cos 3x - K_2 3x\sin 3x + M_2\sin 3x + M_2 3x\cos 3x$$

$$y''_p = -K_1\cos x - M_1\sin x - 3K_2\sin 3x - 3K_2\sin 3x - 9K_2 x\cos 3x + 3M_2\cos 3x + 3M_2\cos 3x - 9M_2 x\sin 3x$$

$$y''_p = -K_1\cos x - M_1\sin x - 6K_2\sin 3x - 9K_2 x\cos 3x + 6M_2\cos 3x - 9M_2 x\sin 3x$$

Substitute in the ODE:

$$3\,(-K_1\cos x - M_1\sin x - 6K_2\sin 3x - 9K_2 x\cos 3x + 6M_2\cos 3x - 9M_2 x\sin 3x) + 27\,(K_1\cos x + M_1\sin x + K_2 x\cos 3x + M_2 x\sin 3x) = 3\cos x + \cos 3x$$

$$-3\,K_1\cos x - 3\,M_1\sin x - 18\,K_2\sin 3x - 27\,K_2 x\cos 3x + 18\,M_2\cos 3x - 27M_2 x\sin 3x + \ 27K_1\cos x + 27M_1\sin x + 27K_2 x\cos 3x + 27M_2 x\sin 3x = 3\cos x + \cos 3x$$

$$\cos x\,(-3\,K_1+27K_1)+\sin x\,(-3\,M_1+27M_1)+18\,M_2\cos 3x-18\,K_2\sin 3x$$
$$+\,x\cos 3x\,(-27\,K_2+27\,K_2)+x\sin 3x\,(-27M_2+27M_2)=3\cos x+\cos 3x$$
$$\cos x\,(24\,K_1)+\sin x\,(24\,M_1)+18\,M_2\cos 3x-18\,K_2\sin 3x+x\cos 3x\,(0)+x\sin 3x\,(0)$$
$$=3\cos x+\cos 3x$$
$$\cos x\,(24\,K_1)+\sin x\,(24\,M_1)+18\,M_2\cos 3x-18\,K_2\sin 3x=3\cos x+\cos 3x$$

Equating the coefficients of cos x, sin x, sin 3x and cos 3x on both sides gives:

$$\begin{cases} 24\,K_1=3 \\ 24M_1=0 \\ 18\,M_2=1 \\ -18\,K_2=0 \end{cases} = \begin{cases} K_1=\dfrac{3}{24} \\ M_1=\dfrac{0}{24} \\ M_2=\dfrac{1}{18} \\ K_2=-\dfrac{0}{18} \end{cases} \Rightarrow \begin{cases} K_1=\dfrac{1}{8} \\ M_1=0 \\ M_2=\dfrac{1}{18} \\ K_2=0 \end{cases}$$

Substituting above values in y_p , we have:

$$y_p=\frac{1}{8}\cos x+0\sin x+0\,x\cos 3x+\frac{1}{18}x\sin 3x$$
$$y_p=\frac{1}{8}\cos x+\frac{1}{18}x\sin 3x$$

Therefore:

$$y=y_h+y_p$$
$$y=A\cos 3x+B\sin 3x+\frac{1}{8}\cos x+\frac{1}{18}x\sin 3x$$

Good Luck