ISRA UNIVERSITY

Islamabad Campus

Semester – Spring 2019

Solution

Calculus-II

Assignment – 5 **Due Date: 11/06/2019**

Marks: 20 **Handout Date: 24/05/2019**

Question # 1:

Solve the following equation:

$$xy'' + 2y' = 0$$

Solution:

The general Euler-Cauchy equation is:

$$x^2y" + axy' + by = 0 \ \rightarrow (1)$$

We have the auxiliary equation:

$$m^2 + (a-1)m + b = 0 \rightarrow (2)$$

$$xy'' + 2y' = 0, to\ make\ it\ general\ multiply\ x\ on\ both\ sides$$

$$x^2y'' + 2xy' = 0 \rightarrow (3)$$

Comparing equation (1) and (3) we have:

$$a = 2\ and\ b = 0$$

Then the auxiliary equation will be:

$$m^2 + (2-1)m + 0 = 0$$

$$m^2 + m = 0$$

$$m(m+1) = 0$$

$$m = 0\ \&\ m + 1 = 0 \Rightarrow m = -1$$

Then the general solution is:

$$y(x) = c_1x^0 + c_2x^{-1}$$

$$y(x) = c_1 + c_2x^{-1}$$

Question # 2:

Solve the Initial Value Problem:

$$x^2y'' + xy' + 9y = 0, \qquad y(1) = 0, y'(1) = 2.5$$

Solution:

The general Euler-Cauchy equation is:

$$x^2y" + axy' + by = 0 \ \rightarrow (1)$$

We have the auxiliary equation:

$$m^2 + (a-1)m + b = 0 \rightarrow (2)$$

$$x^2y'' + xy' + 9y = 0 \rightarrow (3)$$

Comparing equation (1) and (3) we have:

$$a = 1 \; and \; b = 9$$

Then the auxiliary equation will be:

$$m^2 + (1-1)m + 9 = 0$$
$$m^2 + (0)m + 9 = 0$$
$$m^2 + 9 = 0$$

Using Quadratic equation:

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$m = \frac{-0 \pm \sqrt{0^2 - 4(1)(9)}}{2(1)}$$
$$m = \frac{-0 \pm \sqrt{-36}}{2} \Rightarrow m = \pm\frac{i6}{2} = \pm i\,3$$
$$m_1 = +i3 \;\&\; m_2 = -i3$$
$$m = \sqrt{-9} \Rightarrow m = i\,3$$

This is Case # 3 where we have a complex conjugate root, hence the general solution will be:

$$y(x) = x^{\alpha}[Acos(\beta \ln x) + Bsin(\beta \ln x)]$$

Where:

$$\alpha = Real \; part \; and \; \beta = Imaginary \; part$$

Hence:

$$y(x) = x^0[Acos(3\ln x) + Bsin(3\ln x)]$$
$$y(x) = Acos(3\ln x) + Bsin(3\ln x) \rightarrow (4)$$

Now for Particular solution, let’s differentiate equation (4):

$$y'(x) = -\frac{3A}{x}sin(3\ln x) + \frac{3B}{x}cos(3\ln x) \rightarrow (5)$$

Now put the initial value conditions in equation (4) & (5):

$$y(1) = 0, put \; in \; equ(4)$$
$$y(1) = Acos(3\ln 1) + Bsin(3\ln 1)$$
$$0 = Acos(0) + Bsin(0)$$
$$0 = A(1) + B(0) \Rightarrow A = 0$$
$$y'(1) = 2.5, put \; in \; equ \; (5)$$
$$y'(1) = -\frac{3A}{1}sin(3 \; \ln 1) + \frac{3B}{1}cos(3\ln 1)$$
$$2.5 = -3Asin(0) + 3Bcos(0)$$
$$2.5 = -3A(0) + 3B(1) \Rightarrow 2.5 = 0 + 3B$$
$$3B = 2.5 \Rightarrow B = 0.8333$$

Then substituting values of A and B back in equation (4) we have,

$$y(x) = 0cos(3\ln x) + 0.8333sin(3\ln x)$$
$$y(x) = 0.8333 \; sin(3\ln x)$$

Good Luck