



ISRA UNIVERSITY

Islamabad Campus

Semester – Spring 2019

Solution

Calculus-II

Assignment – 3

Marks: 10

Due Date: 07/05/2019

Handout Date: 30/04/2019

Question # 1:

Verify by substitution that $y_1 = e^x$ and $y_2 = e^{-x}$ are solution of the ODE $y'' - y = 0$, solve for the initial value problem $y(0) = 6$ and $y'(0) = -2$.

Solution:

Let's check:

$$\begin{aligned}y_1 &= e^x \\y_1' &= e^x, y_1'' = e^x \\y'' - y &= 0 \\e^x - e^x &= 0 \\0 &= 0, \text{hence proved.}\end{aligned}$$

Now let's check:

$$\begin{aligned}y_2 &= e^{-x} \\y_2' &= -e^{-x}, y_2'' = e^{-x} \\y'' - y &= 0 \\e^{-x} - e^{-x} &= 0 \\0 &= 0, \text{hence proved.}\end{aligned}$$

As y_1 and y_2 are the solutions of the given ODE the general solution is:

$$\begin{aligned}y(x) &= c_1 y_1 + c_2 y_2 \\y(x) &= c_1 e^x + c_2 e^{-x}\end{aligned}$$

Now for particular solution:

$$\begin{aligned}y'(x) &= c_1 e^x - c_2 e^{-x} \\y(0) &= c_1 e^0 + c_2 e^{-0} \Rightarrow 6 = c_1 + c_2 \rightarrow (1) \\y'(0) &= c_1 e^0 - c_2 e^{-0} \Rightarrow -2 = c_1 - c_2 \rightarrow (2)\end{aligned}$$

Now add eq (1) and eq (2):

$$\begin{aligned}c_1 + c_2 &= 6 \\c_1 - c_2 &= -2 \\2c_2 &= 8 \\c_2 &= 4, \text{put in eq (1)} \\c_1 + c_2 &= 6 \Rightarrow c_1 + 4 = 6 \\c_1 &= 2\end{aligned}$$

Hence:

$$y(x) = 2e^x + 4e^{-x}$$