ISRA UNIVERSITY
Islamabad Campus

Semester – Spring 2019

Calculus-II
Solution

Assignment – 2 **Due Date: 02/04/2019**
Marks: 10 **Handout Date: 26/03/2019**

Question # 1:
Solve the given Linear Differential equation of first order and find its Particular solution:

$$y' + \frac{1}{x}y = \sin x\,, \qquad y(\pi) = 1$$

Solution:

The differential equation is already in standard form. Since $p(x) = \frac{1}{x}$ and $r(x) = \sin x$.
The integrating factor is:

$$h = \int p(x)dx = \int \frac{1}{x}dx = \ln x$$

Using:

$$y = e^{-h}\left[\int e^{h} r dx + c\right]$$

$$y = e^{-(\ln x)}\left[\int e^{(\ln x)} \sin x\, dx + c\right]$$

Applying exponent rule: $e^{-(\ln x)} = \left(e^{(\ln x)}\right)^{-1}$

$$y = \left(e^{(\ln x)}\right)^{-1}\left[\int e^{(\ln x)} \sin x\, dx + c\right]$$

Exponential and natural log will cancel out each other then:

$$y = \frac{1}{x}\left[\int x \sin x\, dx + c\right] \rightarrow (1)$$

Applying integration by parts: $\int uv' = uv - \int u'v$

$$\int x \sin x\, dx\,, \qquad u = x \; and \; v' = \sin x$$

$$u' = 1 \; and \;\; v = \int \sin x\, dx = -\cos x$$

Now:

$$\int x\sin x dx = x(-\cos x) - \int 1(-\cos x)dx$$

$$\int x\sin x dx = -x\cos x - \int -\cos x dx$$
$$\int x\sin x dx = -x\cos x + \int \cos x dx$$
$$\int x\sin x dx = -x\cos x + \sin x$$

Now put this back in equ (1) gives:

$$y = \frac{1}{x}[-x\cos x + \sin x + c]$$
$$xy = -x\cos x + \sin x + c, \quad General\ Solution$$

For particular solution lets use Initial condition $y(\pi) = 1$:

$$\pi.1 = -\pi\cos\pi + \sin\pi + c$$
$$\pi.1 = -\pi(-1) + 0 + c$$
$$c = \pi - \pi = 0$$

Hence the final particular solution is:

$$xy = -x\cos x + \sin x \ \ or\ y = \frac{\sin x}{x} - \cos x\,, Particular\ Solution$$

<u>Question # 2:</u>

Solve the given Bernoulli's equation:

$$y' - \frac{1}{x}y = xy^2$$

<u>Solution:</u>

$$y' - \frac{1}{x}y = xy^2 \to (1)$$

Now $a = 2$ then:

$$u = y^{1-a} = y^{1-2} \Rightarrow y^{-1}$$

Now differentiate u:

$$\frac{du}{dy} = -y^{-1-1} = -y^{-2}\ or\ u' = -y^{-2}y'$$

Multiply $-y^{-2}$ with equ. (1):

$$-y^{-2}y' - (-y^{-2})\frac{1}{x}y = (-y^{-2})xy^2$$
$$-y^{-2}y' + \frac{1}{x}y^{-2+1} = -xy^{-2+2}$$
$$-y^{-2}y' + \frac{1}{x}y^{-1} = -xy^0$$
$$-y^{-2}y' + \frac{1}{x}y^{-1} = -x \to (2)$$

Now let put $u' = -y^{-2}y', u = y^{-1}$:

$$u' + \frac{1}{x}u = -x \to Linearform$$

$$p(x) = \frac{1}{x}, r(x) = -x$$

$$h = \int p(x)dx = \int \frac{1}{x}dx = \ln x$$

Then:

$$u = e^{-h}\left[\int e^{h} r dx + c\right]$$

$$u = e^{-lnx}\left[\int e^{\ln x}(-x)\, dx + c\right]$$

$$u = e^{-(\ln x)}\left[\int e^{(\ln x)}(-x)\, dx + c\right]$$

Applying exponent rule: $e^{-(\ln x)} = \left(e^{(\ln x)}\right)^{-1}$

$$u = \left(e^{(\ln x)}\right)^{-1}\left[\int e^{(\ln x)}(-x)\, dx + c\right]$$

$$u = x^{-1}\left[\int x.(-x)\, dx + c\right]$$

$$u = \frac{1}{x}\left[\int -x^2\, dx + c\right]$$

$$u = \frac{1}{x}\left[-\frac{x^3}{3} + c\right]$$

$$u = \frac{1}{x}\left(-\frac{x^3}{3}\right) + \frac{c}{x}$$

$$u = \left(-\frac{x^2}{3}\right) + \frac{c}{x}$$

Now: $u = y^{-1}, y = u^{-1}$

$$y = \left[\left(-\frac{x^2}{3}\right) + \frac{c}{x}\right]^{-1}$$

Good Luck