ISRA UNIVERSITY

Islamabad Campus

Program: BSC
Semester – Spring 2019

Calculus-II
Solution

Assignment – 1 **Due Date: 19/03/2019**
Marks: 10 **Handout Date: 13/03/2019**

Question # 1:

Find the particular solution of the following equation using method of separation of variables:

$$y' = xe^{2y}, \qquad y(0) = -1$$

Solution:

$$\frac{dy}{dx} = xe^{2y}$$

Separating the variables:

$$\frac{1}{e^{2y}}dy = xdx$$
$$e^{-2y}dy = xdx$$

Integrating both sides:

$$\int e^{-2y}dy = \int xdx$$
$$-\frac{1}{2}e^{-2y} = \frac{x^2}{2} + c_1$$
$$e^{-2y} = (-2)\frac{x^2}{2} + (-2)c_1, \qquad \therefore -2c_1 = c$$
$$e^{-2y} = -x^2 + c$$

Taking natural log on both sides:

$$\ln(e^{-2y}) = \ln(-x^2 + c)$$
$$-2y = \ln(-x^2 + c)$$
$$y = -\frac{1}{2}\ln(-x^2 + c), \quad General\ solution$$

Applying initial condition:

$$y(0) = -1$$
$$-1 = -\frac{1}{2}\ln(-0^2 + c)$$

$$2 = \ln(c)$$

$$e^2 = e^{\ln(c)}$$
$$e^2 = c$$

Hence the final particular solution is:

$$y = -\frac{1}{2}\ln(-x^2 + e^2)\,, \quad Particular\ solution$$

Question # 2:

Find the genera solution of the following equation using method of Exact differential equations:

$$(2xy - 3x^2)dx + (x^2 - 2y)dy = 0$$

Solution:

$$(2xy - 3x^2)dx + (x^2 - 2y)dy = 0$$

Check for exactness:

$$M = 2xy - 3x^2, \quad N = x^2 - 2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}[2xy - 3x^2] = 2x\ , \quad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}[x^2 - 2y] = 2x$$

Hence:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

The given differential equation is exact.

The general solution u (x, y) = c is given by:

$$u(x,y) = \int M dx + k(y)$$

$$u(x,y) = \int (2xy - 3x^2)dx + k(y)$$

$$u(x,y) = x^2y - x^3 + k(y)$$

Now to find k (y) lets differentiate the above equation:

$$u(x,y) = \frac{\partial}{\partial y}[x^2y - x^3 + k(y)] = N(x,y)$$

$$u(x,y) = x^2 + k'(y) = x^2 - 2y$$

$$k'(y) = -2y$$

Now integrate k' (y):

$$\int k'(y)\,dy = -\int 2y\,dy$$

$$k(y) = -y^2 + c_1$$

Now:

$$u(x,y) = x^2y - x^3 - y^2 + c_1 = c$$

The general solution is:

$$x^2y - x^3 - y^2 = c$$

Good Luck