ASSIGNMENT #3:

Q#1:

$$y(t) = x(t) * h(t) = ?$$

$$x(t) = 2u(t) \quad , \quad h(t) = 6e^{-t}u(t)$$

Sol:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} [2u(\tau)]\, [6e^{-(t-\tau)}u(t-\tau)]\, d\tau$$

$$= \int_{0}^{t} 12\, e^{-(t-\tau)}\, d\tau = 12\int_{0}^{t} e^{-t+\tau}\, d\tau$$

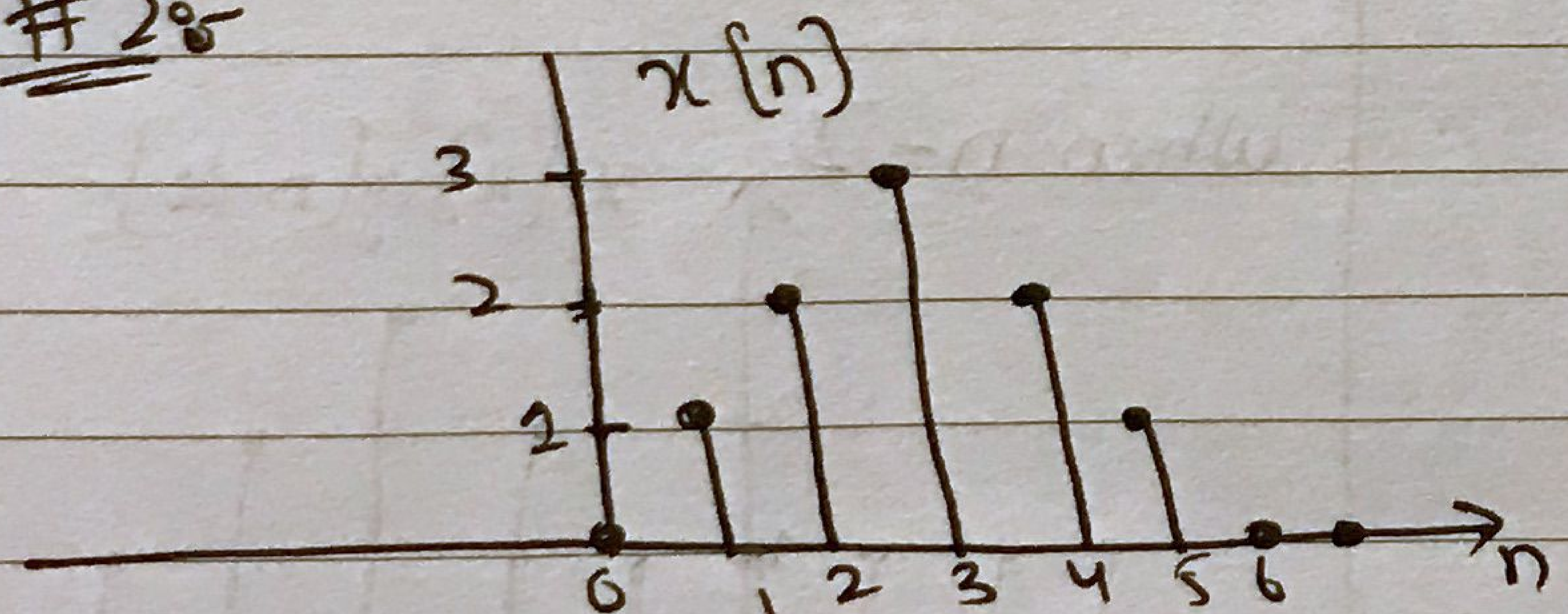
$$= 12e^{-t}\int_{0}^{t} e^{\tau}\, d\tau = 12e^{-t}\left[e^{\tau}\right]_{0}^{t}$$

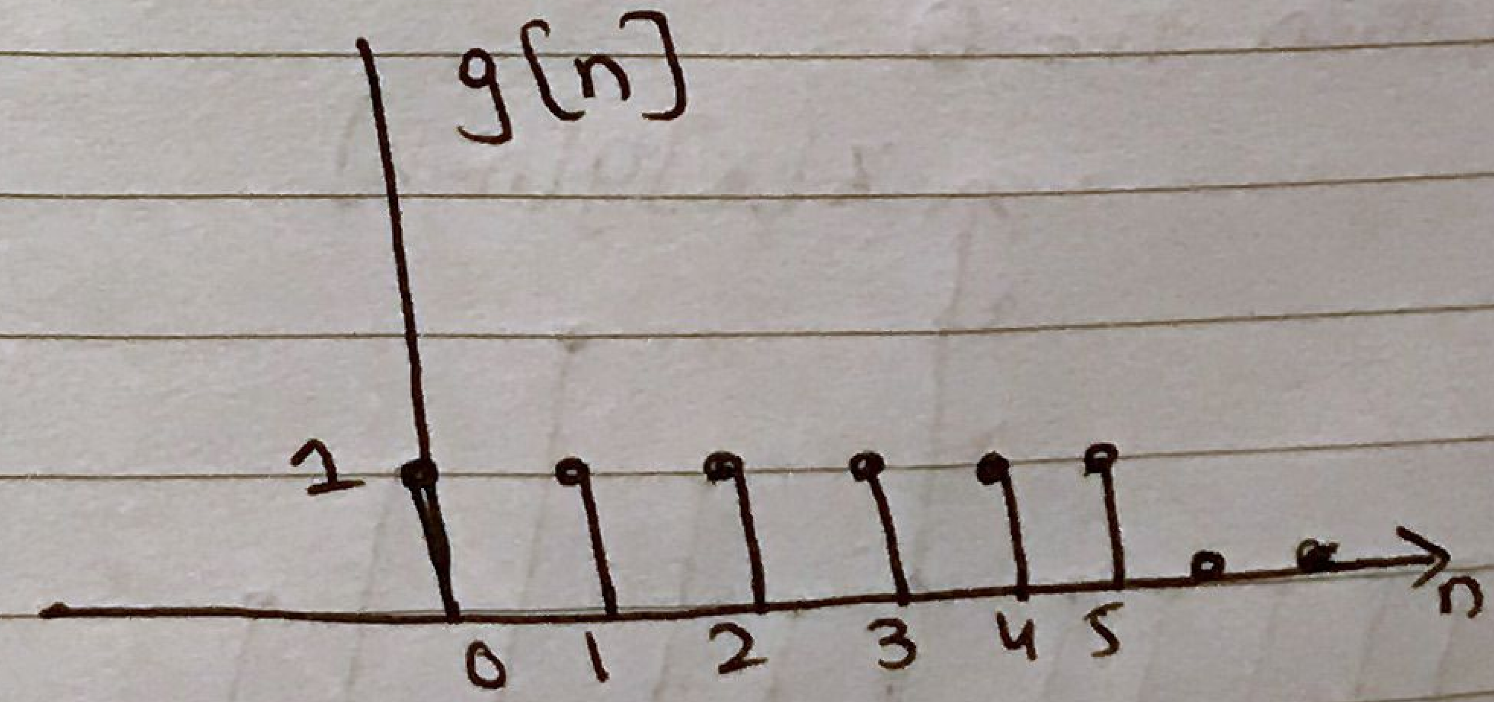
$$= 12e^{-t}\left[e^{t} - e^{0}\right] = 12e^{-t}\left[e^{t} - 1\right]$$

$$= 12\left[e^{-t}e^{t} - e^{-t}\right] \Rightarrow \cancel{12e}$$

$$y(t) = 12\left[1 - e^{-t}\right] u(t)$$

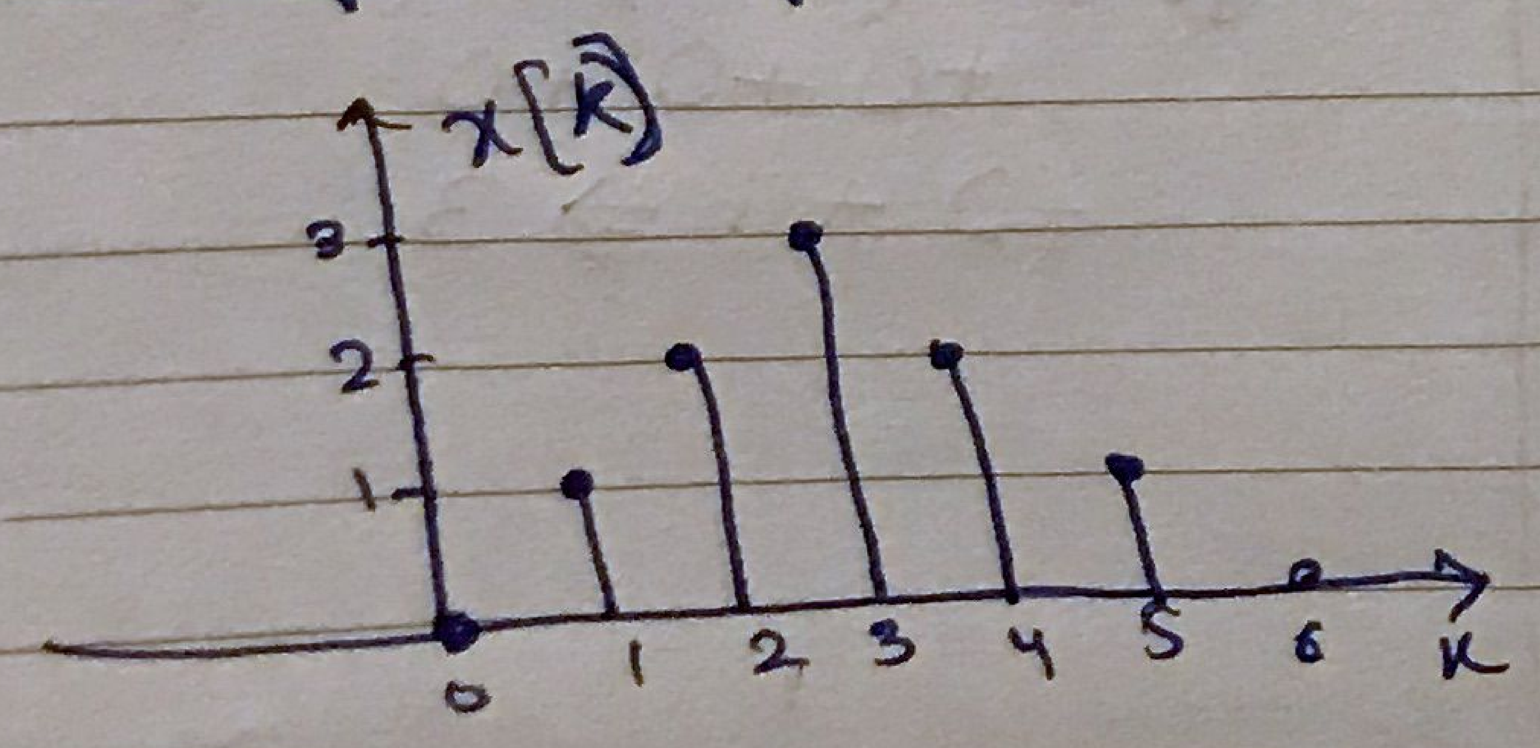
Q#2:

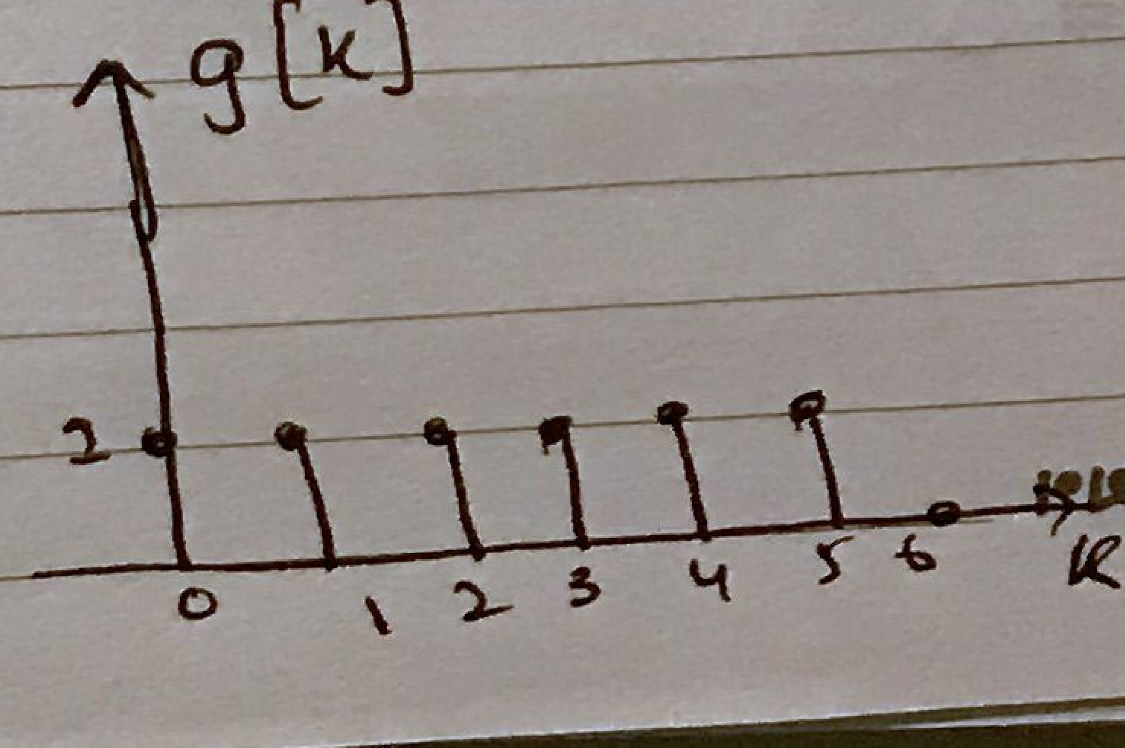




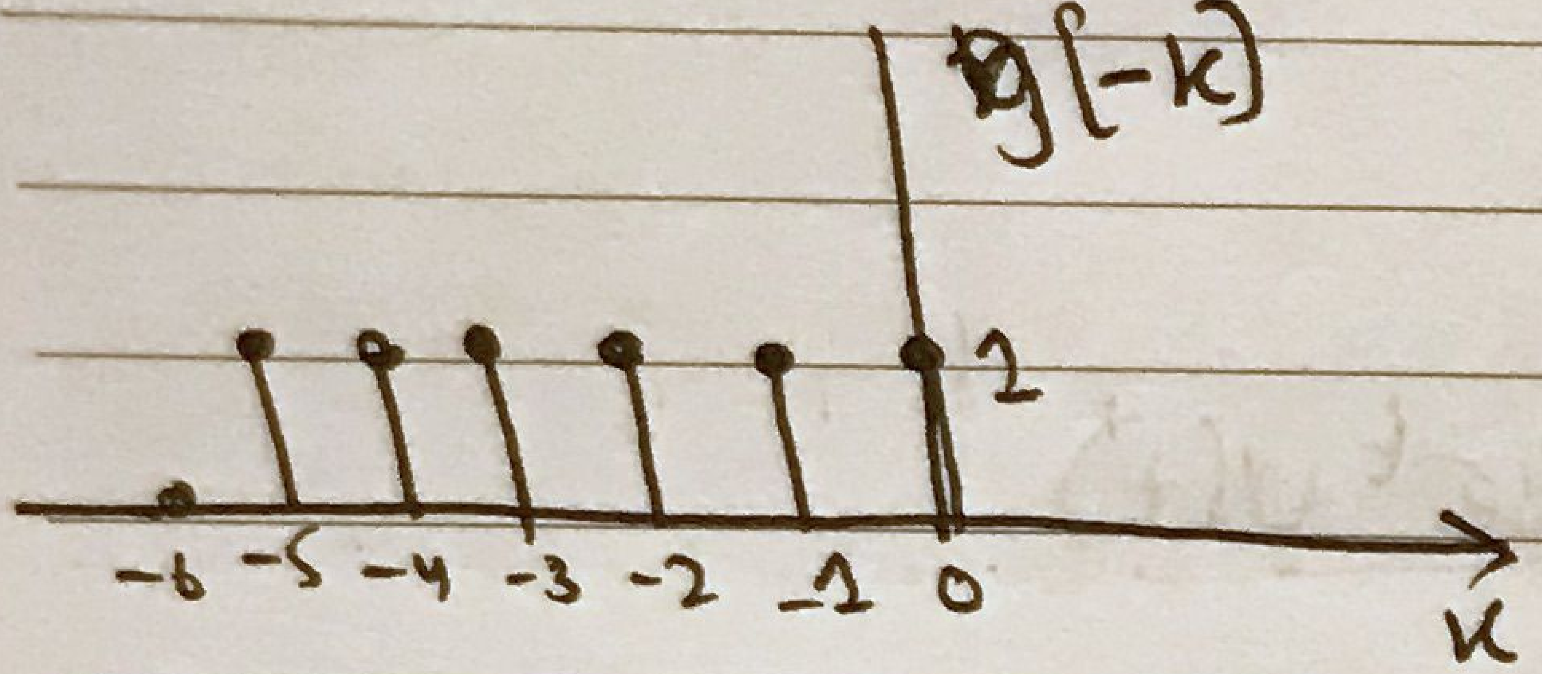
Sol:

Step #1: Replace $n \to k$

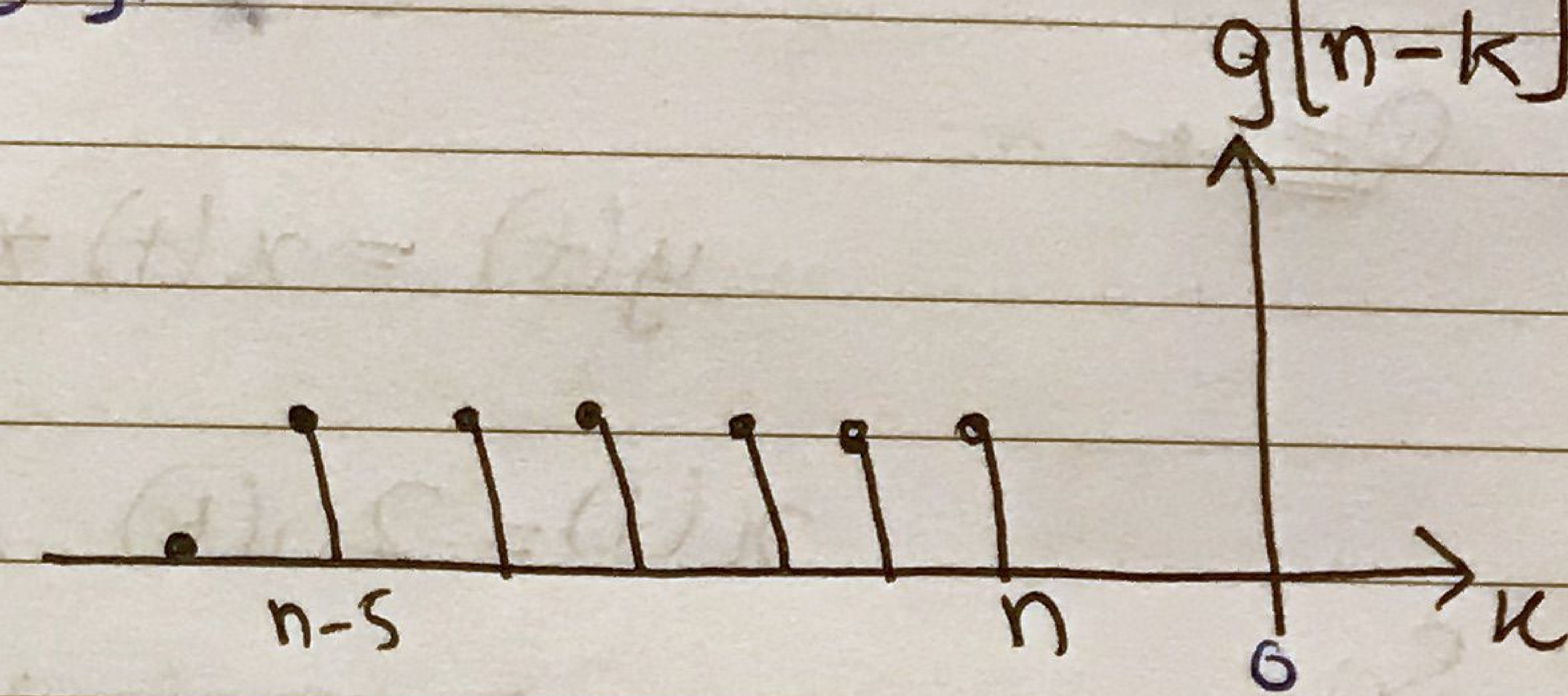




Step #2:- Flip and shift g[k].



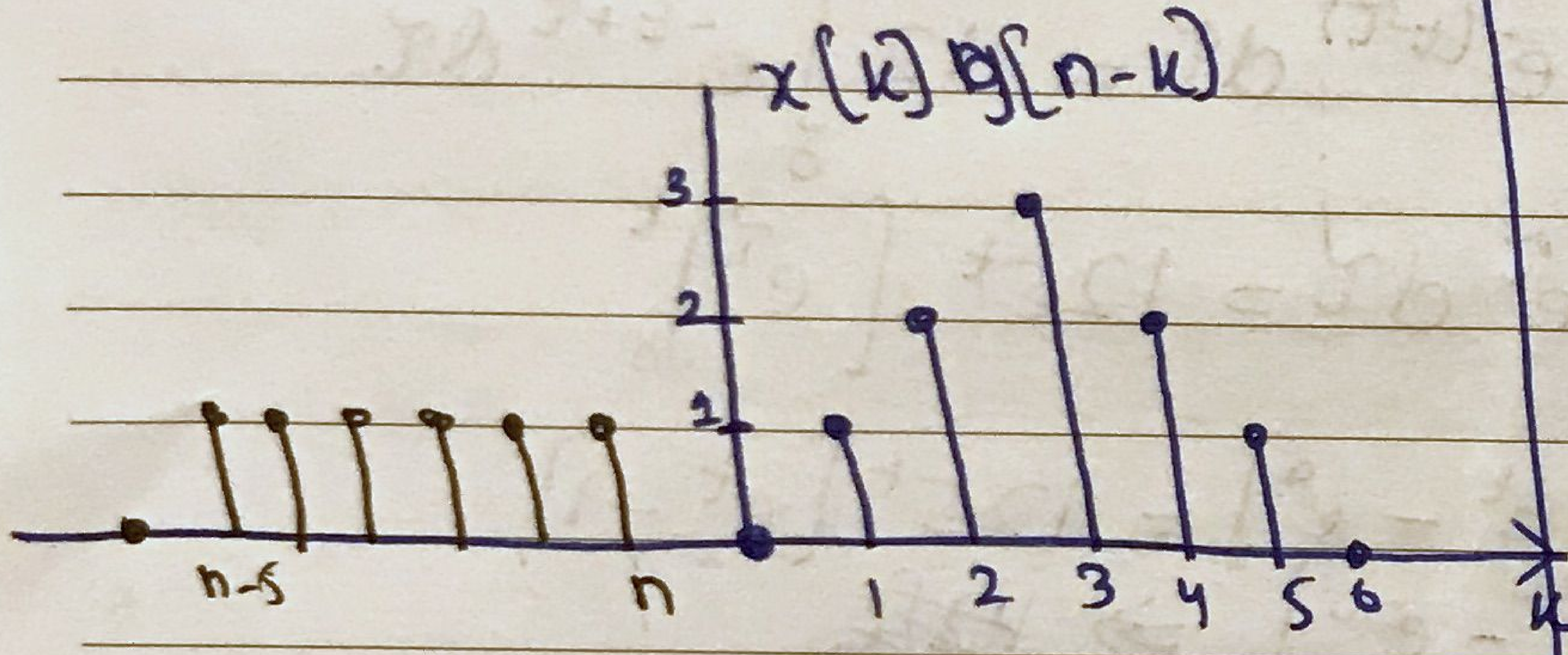
$\Rightarrow$



Step #3:- Now convolve x[k] and g[n-k].

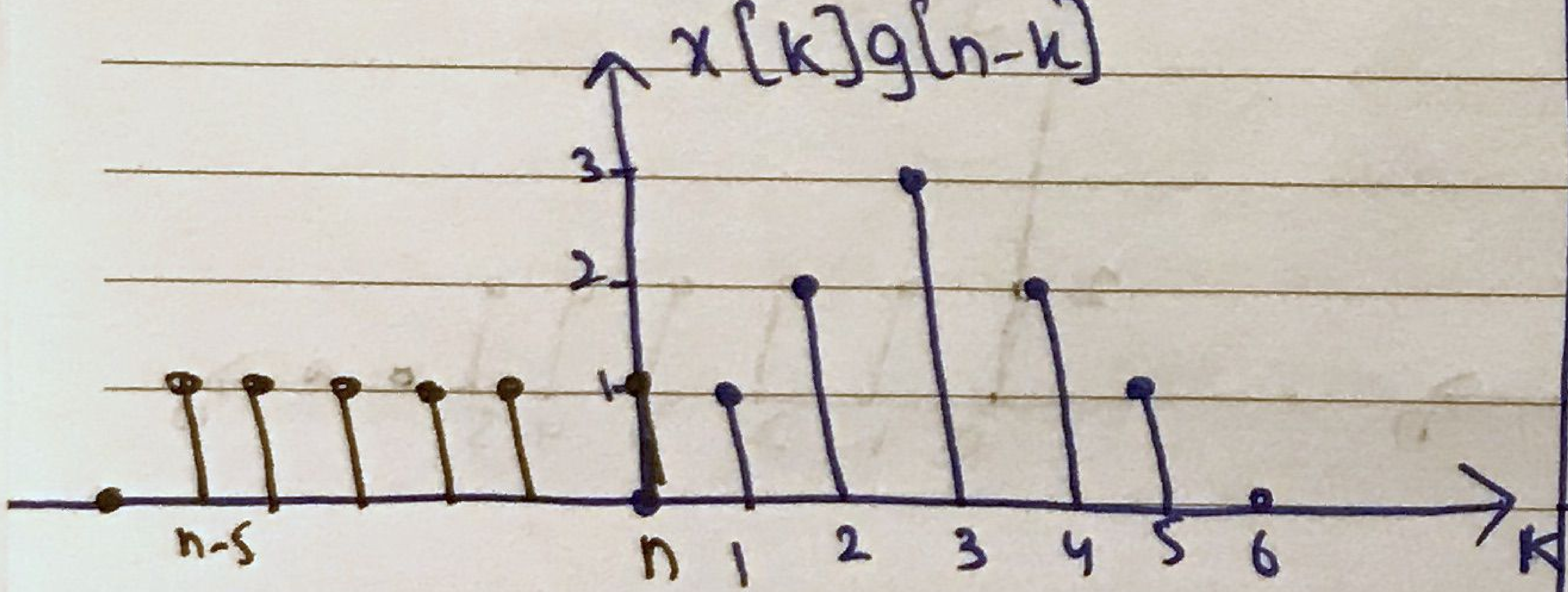
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-k]$$

when $n<0$.



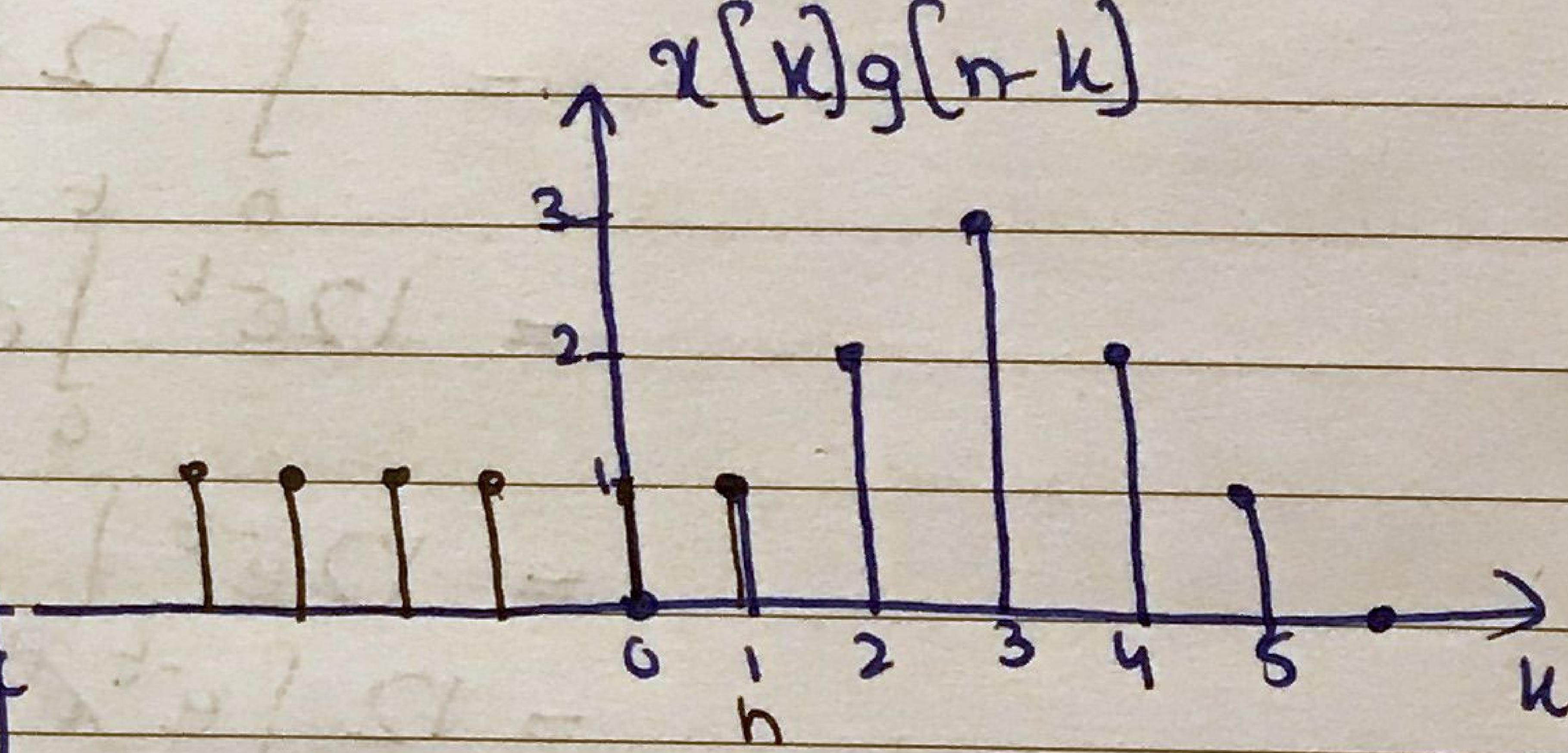
$y[n]=0$, as there is no overlapping.

when $n=0$



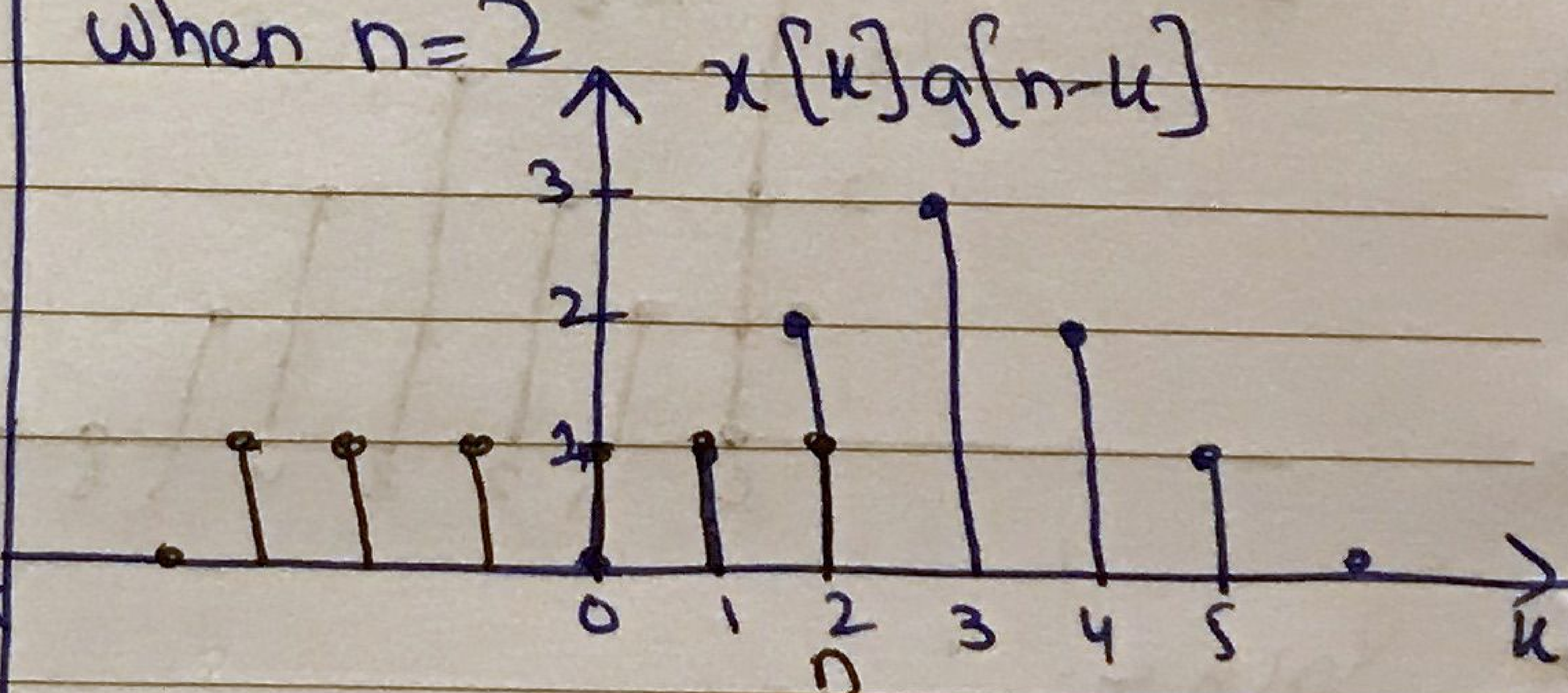
$y[0] = (0\times1)+0+0+0+0+0$
$\Rightarrow 0$

when $n=2$



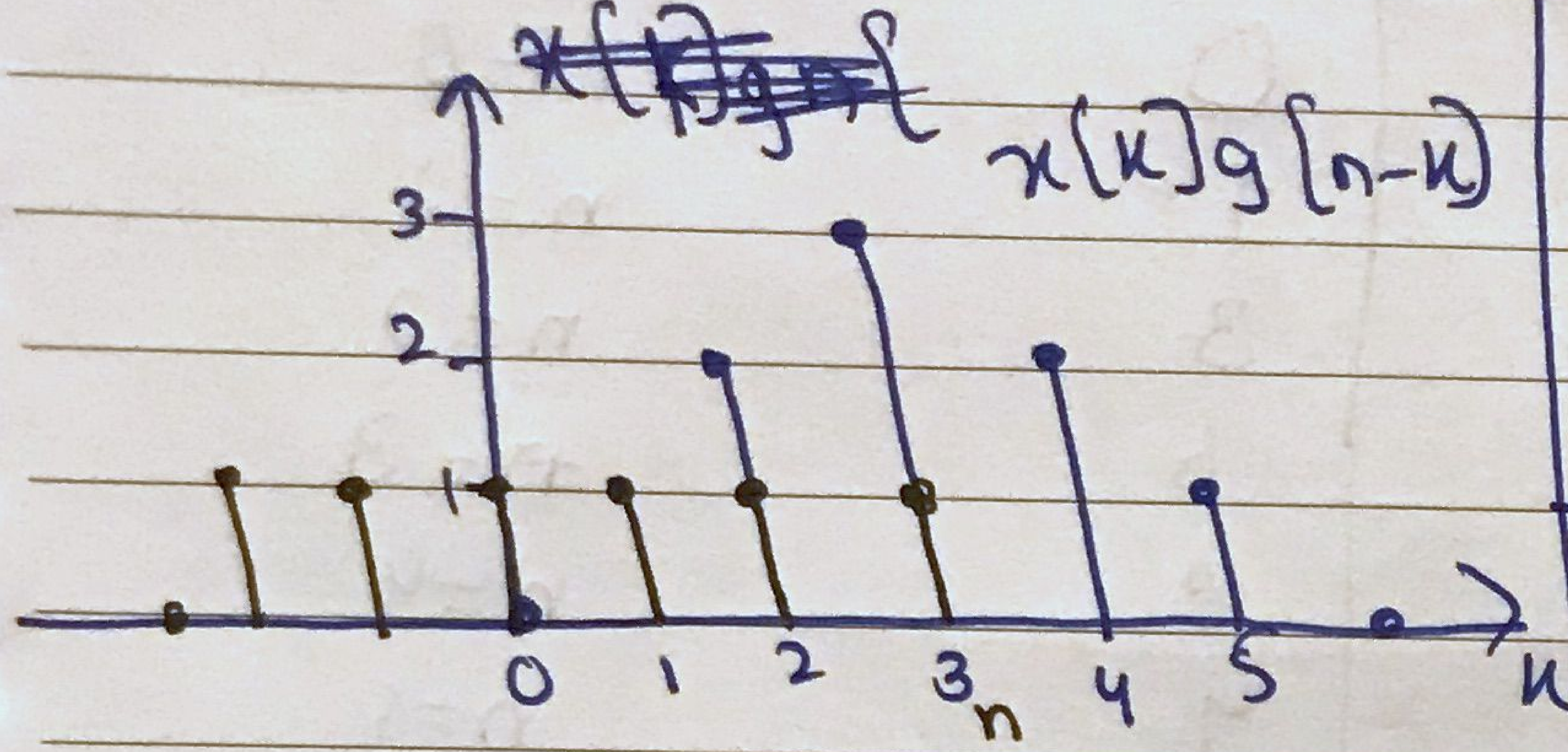
$y[1] = (1\times1)+(1\times0)+0+0+0+0$
$\Rightarrow 1$

when $n=2$



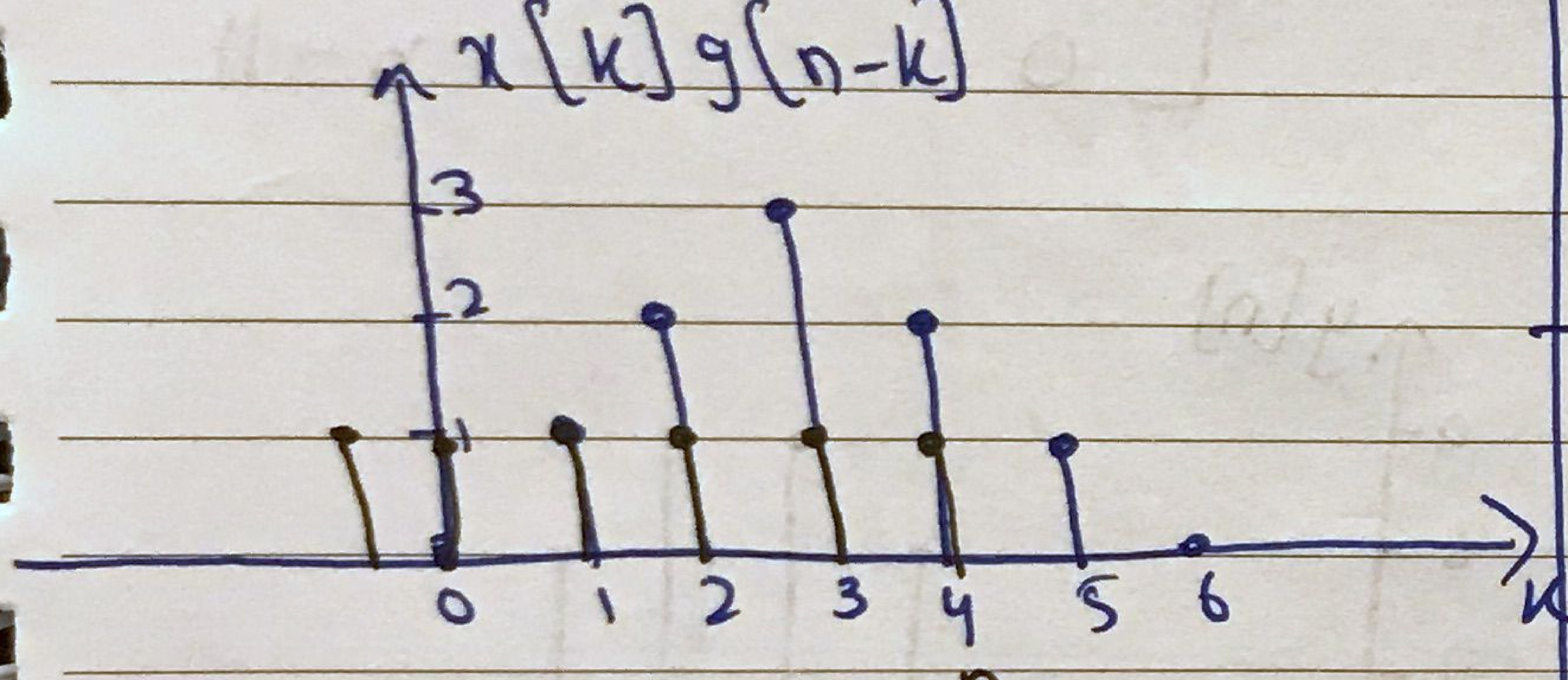
$y[2] = (2\times1)+(1\times1)+(1\times0)$
$+0+0+0$
$=2+1 \Rightarrow 3$

when n=3



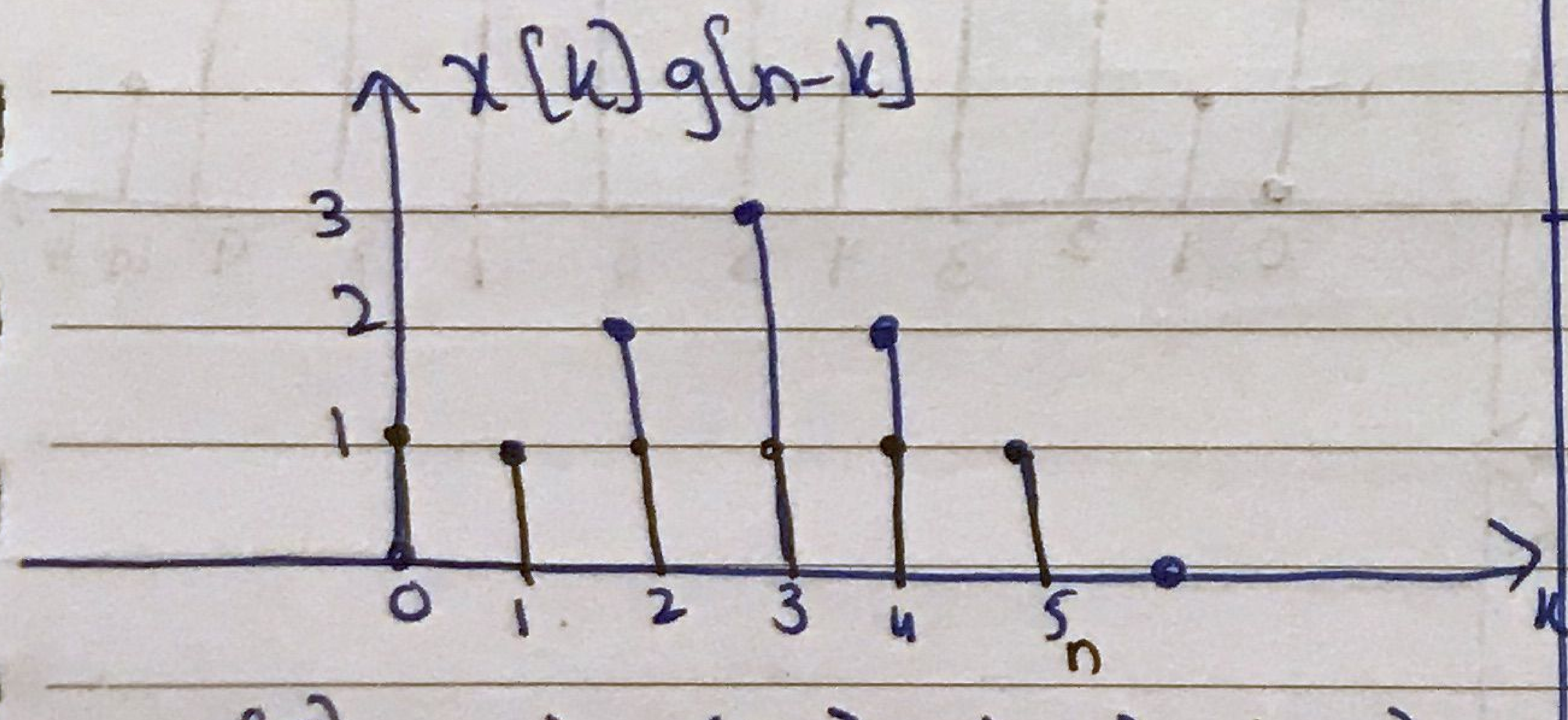
$$y[3] = (3\times1) + (2\times1) + (1\times1) + (1\times0) + 0 + 0$$
$$= 3+2+1 \Rightarrow 6$$

when n=4



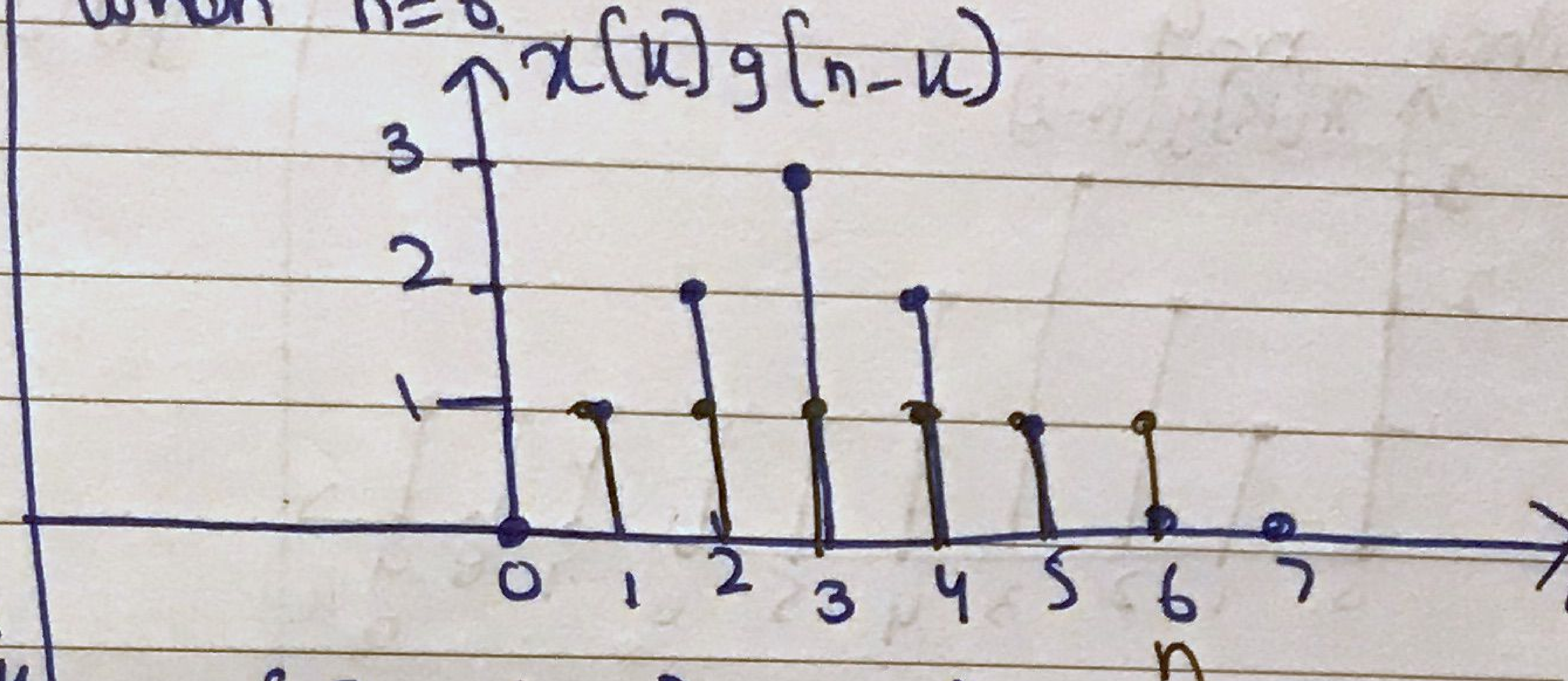
$$y[4] = (2\times1) + (3\times1) + (2\times1) + (1\times1) + (1\times0) + 0$$
$$= 2+3+2+1 \Rightarrow 8$$

when n=5



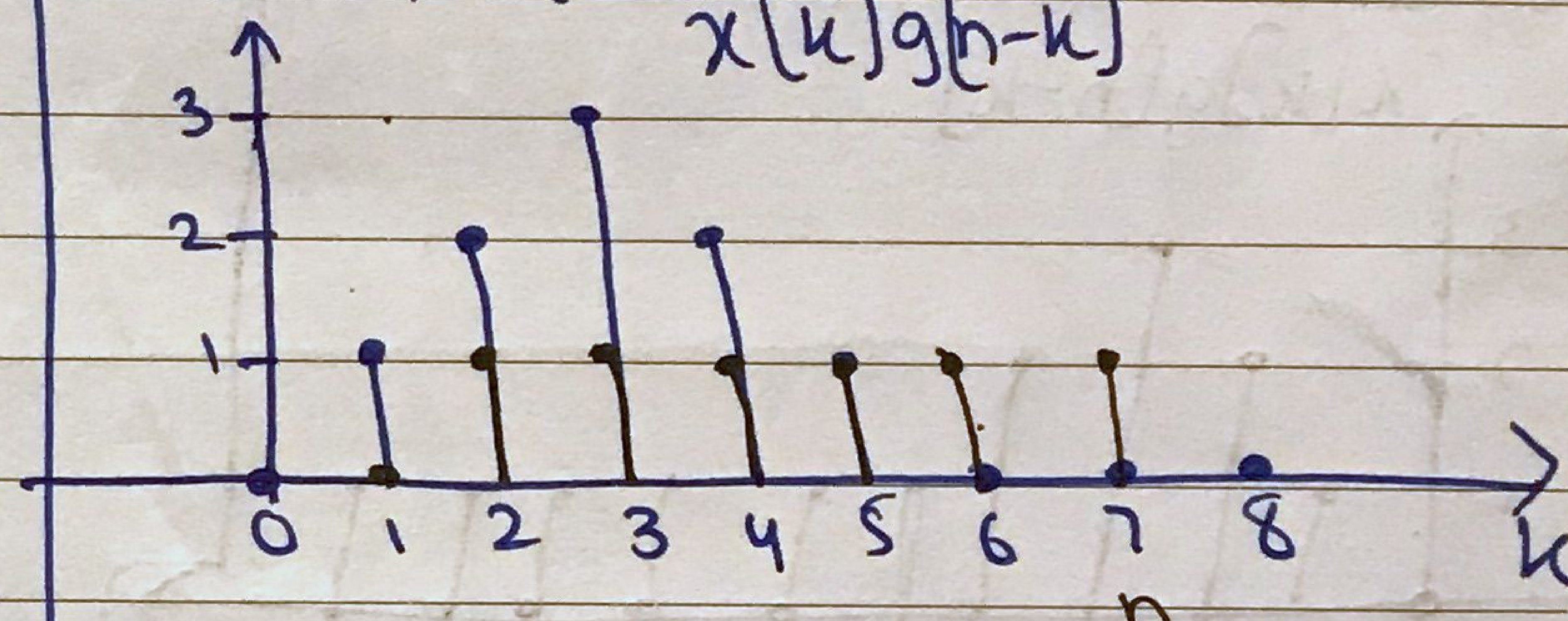
$$y[5] = (1\times1) + (2\times1) + (3\times1) + (2\times1) + (1\times1) + (1\times0)$$
$$= 1+2+3+2+1 \Rightarrow 9$$

when n=6.



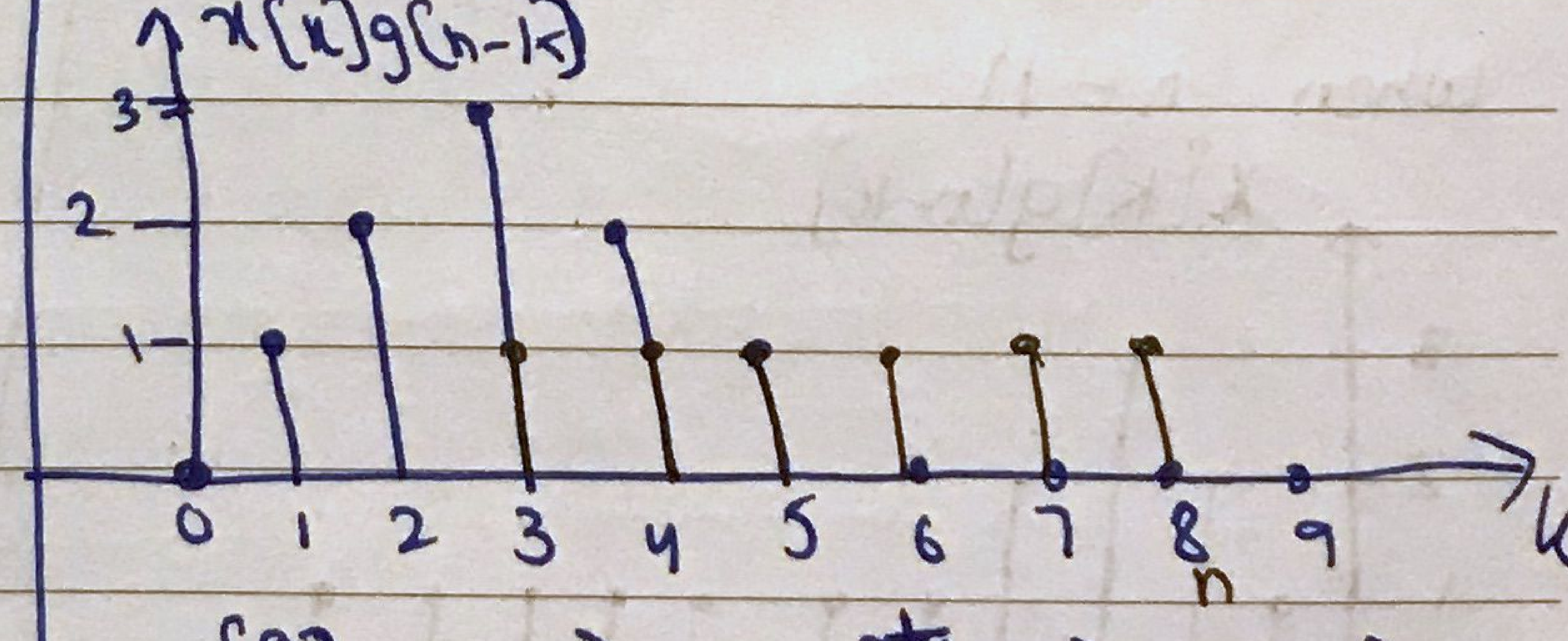
$$y[6] = (0\times1) + (1\times1) + (2\times1) + (3\times1) + (2\times1) + (1\times1)$$
$$= 1+2+3+2+1 \Rightarrow 9$$

when n=7



$$y[7] = (0\times1) + (0\times1) + (1\times1) + (2\times1) + (3\times1) + (2\times1)$$
$$= 1+2+3+2 \Rightarrow 8$$

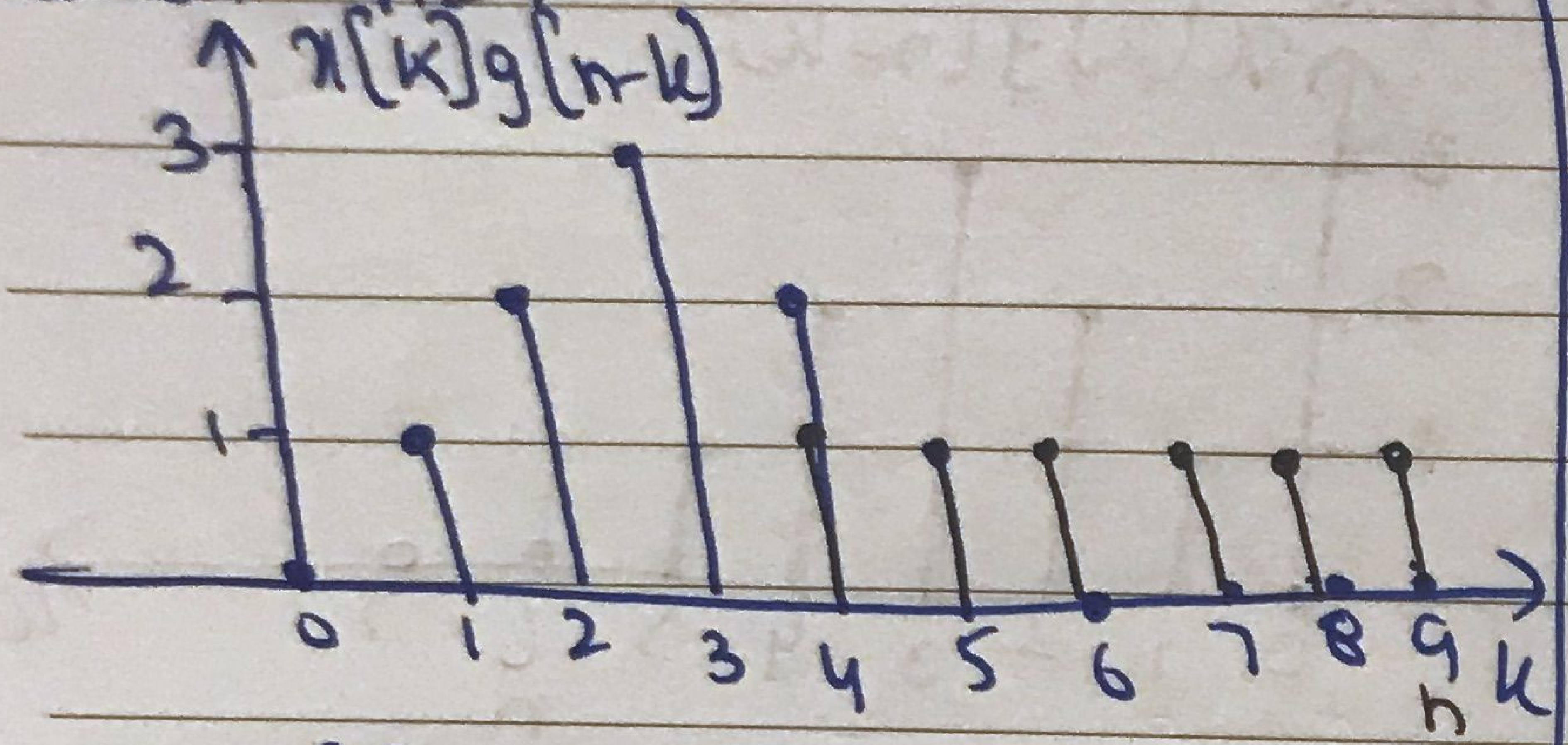
when n=8



$$y[8] = (0\times1) + (0\times1) + (0\times1) + (1\times1) + (2\times1) + (3\times1)$$
$$= 1+2+3 \Rightarrow 6$$

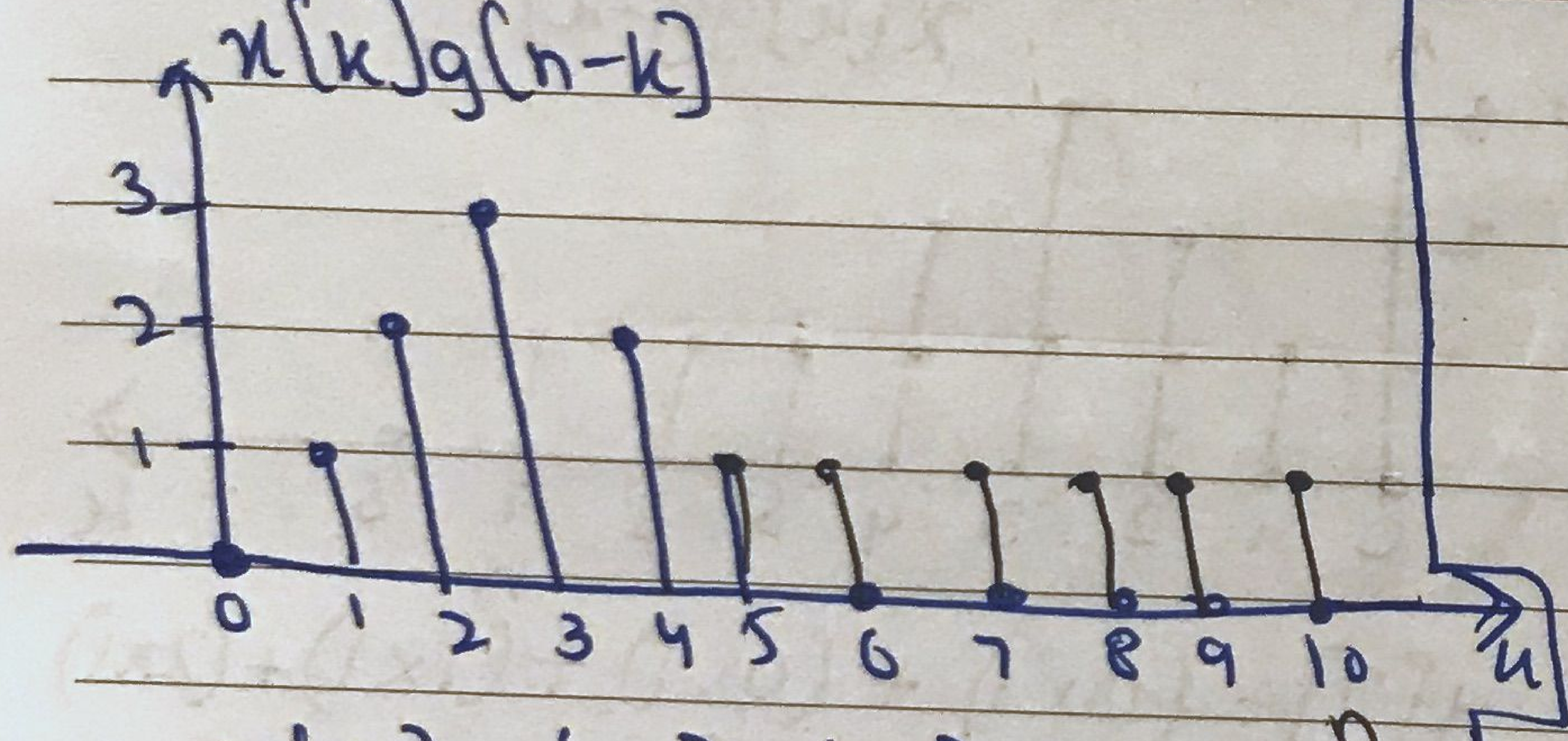
PAPERWORK

when $n=9$



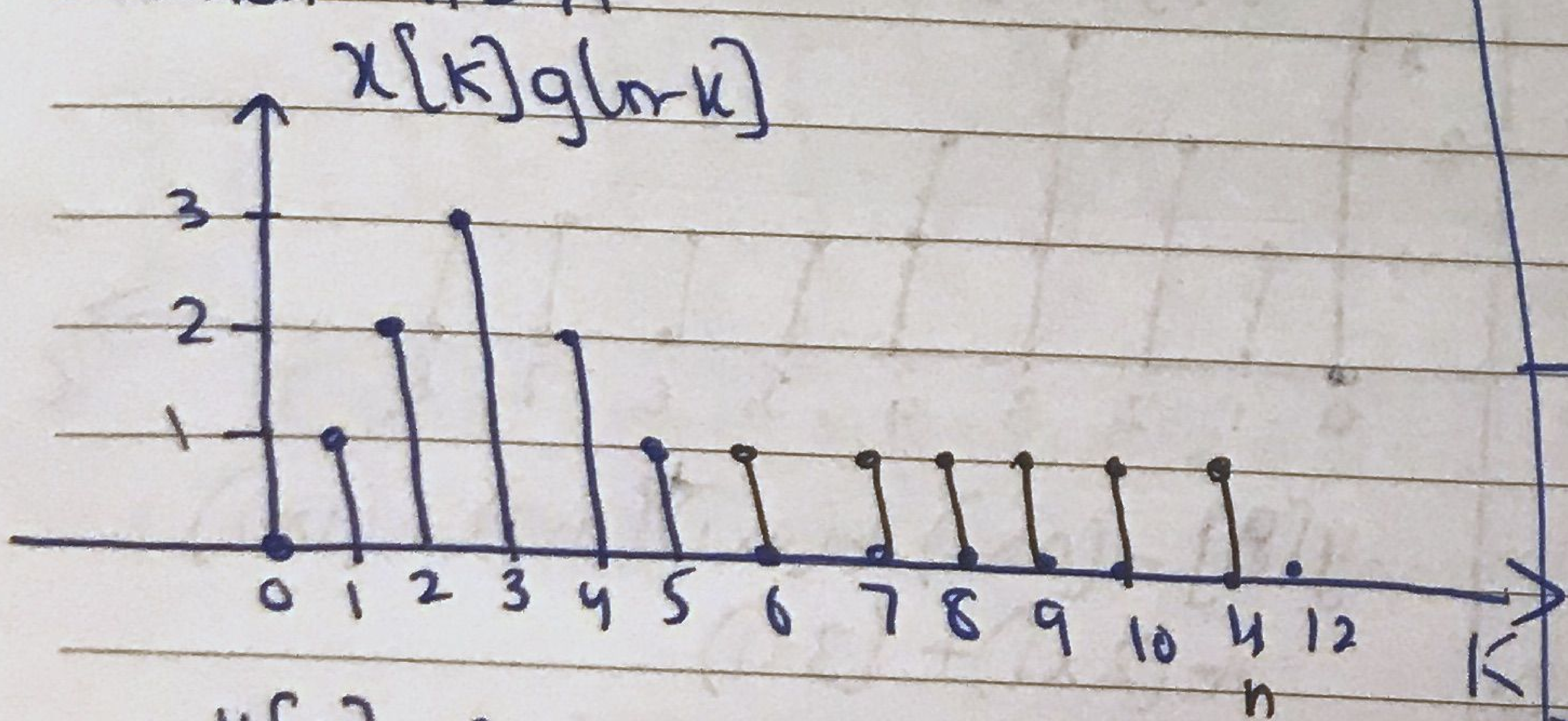
$$y[9] = (0\times1)+(0\times1)+(0\times1)+(0\times1)+(1\times1)+(2\times1)$$
$$= 2+1 \Rightarrow 3$$

when $n=10$



$$y[10] = (0\times1)+(0\times1)+(0\times1)+(0\times1)+(0\times1)+(1\times1)$$
$$y[10] \Rightarrow 1$$

when $n=11$



$y[11]=0$, as no overlapping.

$$y[n] = \begin{cases} 0 & n<0 \\ 0 & n=0 \\ 1 & n=1 \\ 3 & n=2 \\ 6 & n=3 \\ 8 & n=4 \\ 9 & n=5 \\ 9 & n=6 \\ 8 & n=7 \\ 6 & n=8 \\ 3 & n=9 \\ 1 & n=10 \\ 0 & n=11 \end{cases}$$